Indexes, Measures and Subjective Values

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Outline

Measurement

What is?

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More formal

From empirical evidence (ordered structures) to sets of numbers.

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If x is 50kg and y is 100kg, is y twice more heavy than x?

If x is 20^{o}C and y is 40^{o}C, is y twice more hot than x?

Meaningfulness

- Information equivalent numerical scales
- Admissible transformations of numerical scales (create information equivalent representations)
- A class of admissible transformations univocally determines a scale type.

Measurement Scales

- Ordinal Scales
  (strictly increasing transformations)
- Interval Scales
  (positive affine transformations: $\varphi(x) = \alpha x + \beta$)
- Ratio Scales
  (positive homothetic transformations: $\varphi(x) = \alpha x$)
- Absolute Scales
  (identity transformations)

Is this sufficient?

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NO!!

Measures need to be useful.

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x, y, z being the three dimensions of a solid

$x + y + z/3$ is the arithmetic mean, meaningful, but useless
xyz is the geometric mean, meaningful and useful

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Racial statistics

are meaningful and (perhaps) useful, but in many places are not legitimated, if not forbidden.

What do we do with measures?

1. We create “indexes” which synthesise information and provide systematic insight.
2. Insight is usually used within a decision process for some decision making purpose: create evidence.
3. Critical question: how do we move from an index to a justified decision?

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Human Development Index

$$\text{HDI} = \frac{\text{LEI} + \text{EAI} + \text{GDPI}}{3}$$

$$\text{LEI} = \frac{\text{life expectancy at birth} - 25}{85 - 25}$$

$$\text{EAI} = \frac{2\text{ALI} + \text{ERI}}{3}$$

Human Development Index

$$\text{GDPI} = \frac{\text{transformed income} - W(100)}{W(40\,000) - W(100)}$$

where $W(x)$ represents the conversion of the GDP in standard monetary equivalents (USD) following Atkinson's formula.

Scale Normalisation

	life expectancy	EAI	GDPI
South Korea	71.5	.93	.97
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If the scale is [85,25] then HDI(SK)>HDI(CR)
If the scale is [80,25] then HDI(CR)>HDI(SK)

Compensation

	life expectancy	ALI	ERI	real GDP	HDI
Gabon	54.1	.63	.60	3 641	.56
Solomon Islands	70.8	.62	.47	2 118	.58

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A year of life is equivalent to 100.9 USD(equivalent).
If we transform this equivalent in real USD then poor's people life is less worth than rich people life!

Dimension Independence

	life expectancy	ALI	ERI	real GDP	HDI
x	30	.80	.65	500	.30
y	30	.35	.40	3 500	.34
w	70	.80	.65	500	.52
z	70	.35	.40	3 500	.56

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$y \succ x$, but should also $z \succ w$?

The Air Quality index

pollutant	CO_2	SO_2	O_3	dust
t_1	3	3	8	8
t_2	1	3	8	2
t_3	7	7	7	7

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For the ATMO index t_3 is better than t_2. Is this legitimating (and legitimated)?

Construction of sense

Fact

95% of rural households in XX do not have access to fresh tap water

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Who cares about what and why?

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We can be rigourous without being quantitative. The key concept is meaningfulness.

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Claim 3

Decisions are not in the date, but in the values. Data are necessary, but not sufficient.

What is evaluation?

Measuring values (of decision makers, voters, customers ...)

What is the empirical evidence for value measurement?

- revealed preferences from customers' behaviour in markets
- subjective preferences from direct or indirect observation

Can we measure subjective preferences?

Yes!!

Empirical evidence: preference statements

- direct approach: indifference swaps;
- indirect approach: value estimation through learning algorithms.

Numerical Representations

Weak Orders

If $\succeq$ is a w.o. $\Leftrightarrow$ $\exists f : A \mapsto \mathbb{R} : x \succeq y \leftrightarrow f(x) \geq f(y)$

Interval Orders

If $\succeq$ is an i.o.
$\Leftrightarrow$ $\exists f, g : A \mapsto \mathbb{R} : f(x) > g(x); x \succeq y \leftrightarrow f(x) \geq g(y)$

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Is this sufficient?

NOT always!

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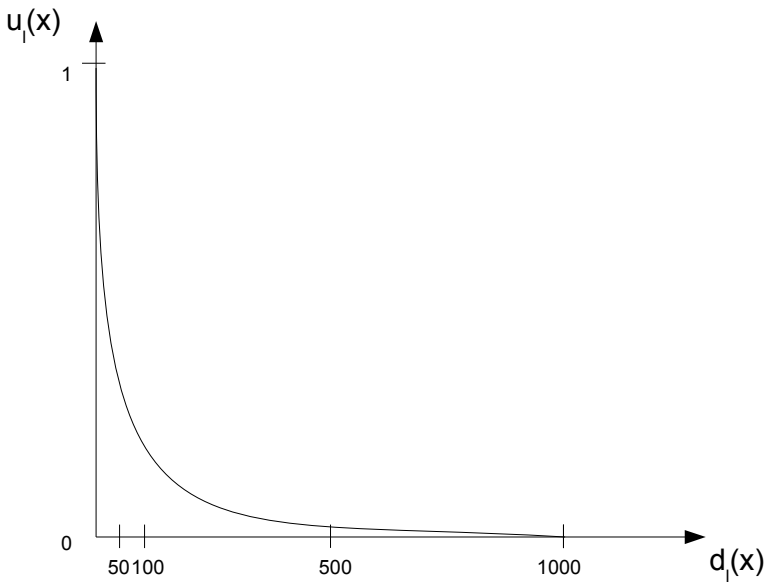
We may need something more rich. We may need to know, when we compare x to y (and we prefer x) if this preference is "stronger" to the one expressed when comparing (on the same dimension) z fo w.

We need to compare differences of preferences

A Value function



For instance, if the above function represents the value of “land use” it is clear that the difference between 50sqm and 100sqm is far more important from the one between 500sqm and 1000sqm.

Is all that sufficient?

NOT always!

If A is described on multiple attributes

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NOT always!

If *A* is described on multiple attributes

1. The problem is that we need to be able to compare the differences of preferences on one dimension to the differences of preferences on another one (let's say differences of preferences on land use with differences of preferences on esthetics.
2. At the same time we need to take into account the intuitive idea that for a given decision maker certain dimensions are more "important" than other ones.

Is it easy?

No!!

Modelling always demand an effort

- the direct approach is cognitively demanding;
- the indirect approach is computationally complex and implies restrictive hypotheses.

Lessons learned

- Income is not reliable as a poverty measure.
- People may feel poor and not be identified as such.
- People may be identified as poor and not feel as such.
- We are more or less poor and in many different ways.

Why do we measure poverty?

Poverty is a problem

For whom is it a problem? Why is it a problem? What does it mean fighting poverty?

Poverty reduction policies

Who is expected to benefit from such policies? How they should benefit? Is that specific policy efficient?

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Endowments and Commodities

Each household is endowed by a set of resources. Endowments allow to produce commodities which can be exchanged within the society.

	Commodities				
Households	m_1	m_2	m_3	$\cdots$	m_m
x_1	0	1	500	$\cdots$	0
x_2	1	0	1000	$\cdots$	0
x_3	0	2	900	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
x_n	0	1	400	$\cdots$	1

Clustering

Households can thus be clustered according to “similar” distribution of commodities. Considering the population X we obtain clusters L_h described by a vector of commodities:

$$L_h = \langle M_{1j}^h, \cdots M_{mj}^h \rangle$$

where:

$$M_{ij} = \frac{|\{x \in L_h : m_i(x) = j\}|}{|\{L_h\}|}$$

Such clusters only identify segments of the population being "similarly poor"

From Commodities to Functionings

We establish a set of potential dimensions of welfare:

- Housing
- Education/Culture
- Nutrition
- Health Care
- Mobility

We then associate subsets of commodities to dimensions of welfare. For instance “house”, “sanitation” and “water” to “Housing”. Some commodities (such as salary) are instead associated to the whole set of welfare dimensions. We call the later “generic commodities”.

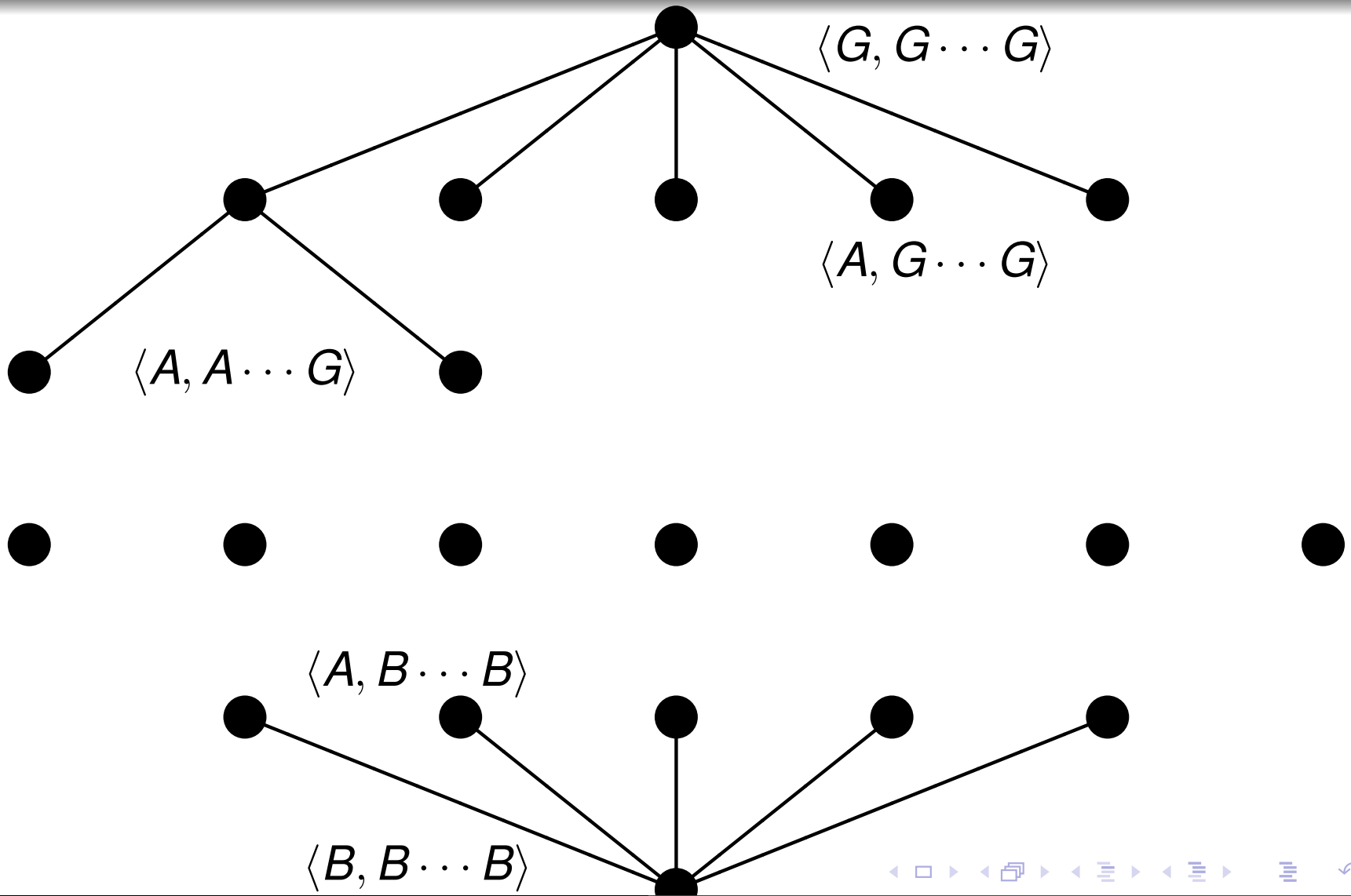
Functionings

	Functionings								
House		0			1			2	
Water			0			1			
Sanitation	0			1			2		3
Housing		B			A			G	

Bad, **A**verage and **G**ood being the three possible functionings of the Housing welfare dimension. We end getting the functionings vector of cluster L_h:

$$F(L_h) = \langle f_1(L_h), \cdots f_t(L_h) \rangle$$

The Functionings Lattice



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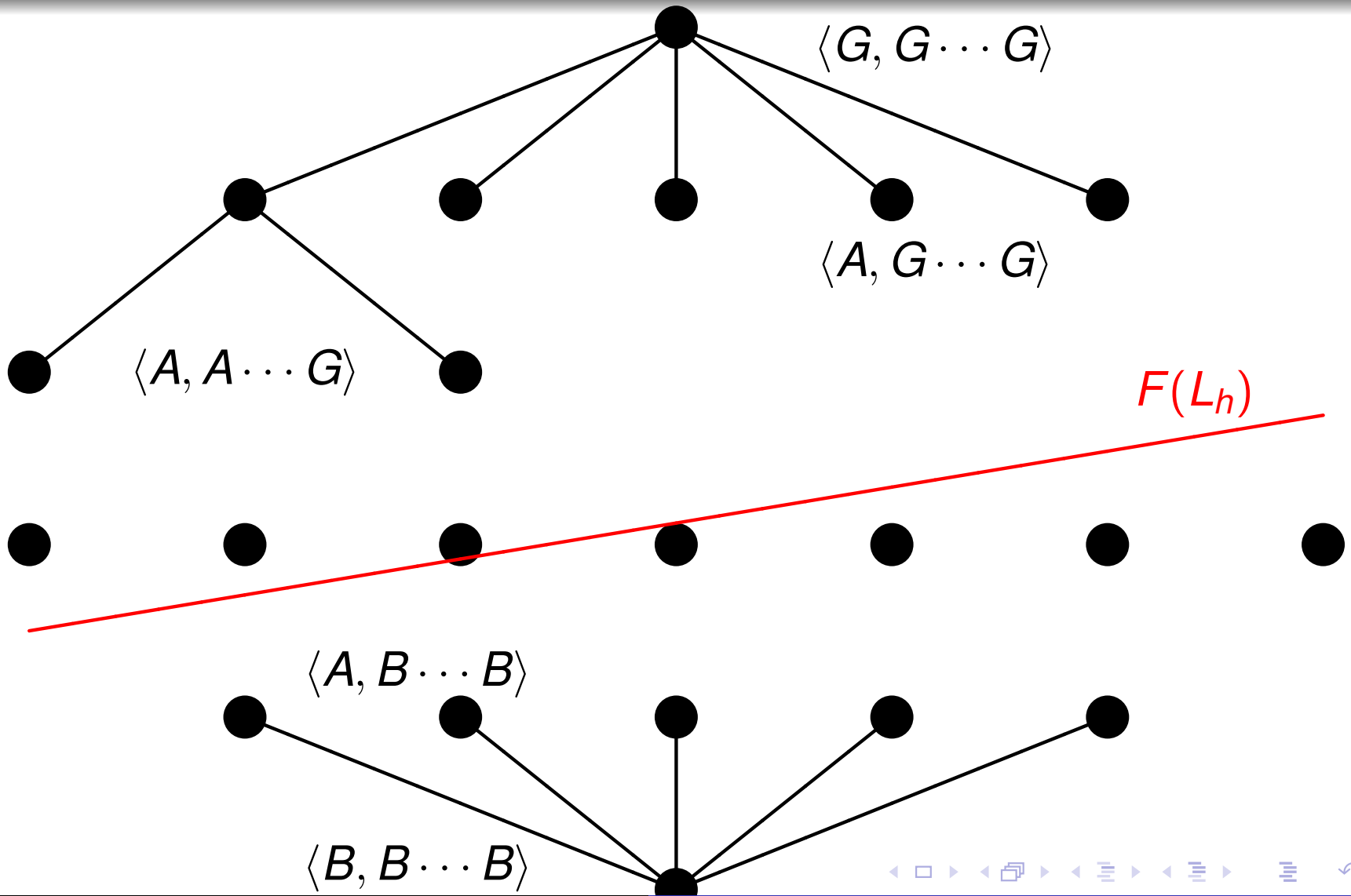

Capabilities

Feasible Capabilities

The subset of functionings compatible with a given set of commodities (resulting from the functionings lattice) establishes the feasible capabilities.

Extended Capabilities

Feasible capabilities can be extended using the generic commodities which allow to increase some of the attainable functionings.

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Policies

A poverty reduction policy should imply:

- Establish the clusters of households.
- Construct a capabilities ordering of the households' clusters.
- Allocate resources to each cluster.
- Monitor the evolution of each cluster position in subsequent orderings constructer at regular time intervals.